Implicit Differentiation

Explicit functions are functions where one variable can be expressed in terms of the other and we can differentiate term by term e.g. $y = 5\ln x + 3x$ differentiates to $\frac{dy}{dx} = \frac{5}{x} + 3$.

An implicit function is one were it is difficult to express one variable in terms of the other i.e. difficult to make y the subject.

We still differentiate term by term, but apply the chain rule to 'y' terms,

i.e. differentiate any 'y' terms by applying the usual differentiation rules, but tack on the $\frac{dy}{dx}$ notation after its derivative.

Example 1: Use implicit differentiation to find $\frac{dy}{dx}$ of $x^2 + y^2 = 9$.

$x^2 + y^2 = 9$ differentiates to $2x + 2y\frac{dy}{dx} = 0$

Rearrange to make $\frac{dy}{dx}$ the subject: $\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

Example 2: In some cases we need to apply the product rule as well.

$4x + 2xy = x^2$ needs to have the product rule applied to the term $2x \cdot y$
[remember if $y = f \cdot g$, then $y' = f' \cdot g + g' \cdot f$]

$4 + (2 \cdot y + \frac{dy}{dx} \cdot 2x) = 2x$

Rearrange to make $\frac{dy}{dx}$ the subject: $\frac{dy}{dx} = \frac{2x-2y-4}{2x} = \frac{x-y-2}{x}$

Example 3: Use implicit differentiation to find $\frac{dy}{dx}$ of $3x + 2x^2 - 3y^2 = 7y$.

$$3 + 4x - 6y\frac{dy}{dx} = 7\frac{dy}{dx}$$

$$7\frac{dy}{dx} + 6y\frac{dy}{dx} = 3 + 4x$$

$$\frac{dy}{dx}(7 + 6y) = 3 + 4x$$

$$\frac{dy}{dx} = \frac{3+4x}{7+6y}$$

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Equations of Tangents and Normals

1) Use implicit differentiation to calculate the equation of the tangent to the circle $x^2 + y^2 = 100$ at the point (-8, 6).

$x^2 + y^2 = 100$ differentiates to $2x + 2y \cdot \frac{dy}{dx} = 0$.

Rearrange to make $\frac{dy}{dx}$ the subject: $\frac{dy}{dx} = \frac{-x}{y}$

$m = \frac{dy}{dx} = \frac{-x}{y}, \quad m(-8, 6) = \frac{-(-8)}{6} = \frac{4}{3}$

Eqn of tangent: $y - 6 = \frac{4}{3}(x - (-8))$

$$y - 6 = \frac{4}{3}x + \frac{32}{3}$$

$$y = \frac{4}{3}x + \frac{50}{3}$$

2) Differentiate $xy^2 + x = 4$, and calculate the equation of the normal at the point (2, 1).

$x \cdot y^2 + x = 4$ differentiates to $\left(1 \cdot y^2 + 2y\frac{dy}{dx} \cdot x\right) + 1 = 0$,

remembering to use the product rule when differentiating $x \cdot y^2$ implicitly.

Rearrange to make $\frac{dy}{dx}$ the subject: $\frac{dy}{dx} = \frac{-y^2-1}{2xy}$

$m(2, 1) = \frac{-1^2-1}{2(2)(1)} = \frac{-2}{4} = \frac{-1}{2}$

$m_n(2, 1) = \frac{-1}{-1/2} = 2$

Eqn of normal: $y - 1 = 2(x - 2)$

$$y - 1 = 2x - 4$$

$$y = 2x - 3$$

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